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A THEORETICAL INVESTIGATION INTO SOME
ASPECTS OF WET COMPRESSION

MAYNARD H. DIXON

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A Theoretical Investigation Into Some Aspects Of Wet Compression

A Thesis

by

Maynard H. Dixon, Lieutenant Commander, U. S. N.

Submitted in partial fulfillment of the requirements for the degree of Master of Science at Rensselaer Polytechnic Institute, Troy, New York.

June 1947

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WET COMPRESSION

Introduction

One of the aims of any power cycle is to compress the working medium through the necessary pressure ratio with a minimum output of mechanical work. This is especially true of the gas turbine cycle, where the net output is roughly the difference between the work output of the turbines and the work required for compression.

The general energy equation applies to portions of any cycle and furnishes a convenient starting point for an investigation into methods of reducing compressor work as a means of increasing the net work available from a gas turbine cycle. The general energy equation may be written as follows:

$$dL + dQ = C_v dT + \frac{1}{J} V dP + \frac{1}{J} P dV + \frac{1}{Jg} v dv \quad \ldots\ldots\ldots\ldots (1)$$

In words, the mechanical work (dL) is that required to take one pound of the working medium at a temperature (T), velocity (v), pressure (P) and specific volume (V) and deliver it to conditions (T + dT), (v + dv), (P + dP) and (V + dV) while an amount of heat (dQ) is added by heat transfer or release.

If, for the purpose of simplification, we can assume that our compressor is perfectly insulated (a condition which can be approached but never attained), with negligible error we can say that (dQ) equals zero. Also, even though there are appreciable inlet and discharge velocities, the pressure ratios reported for continuous flow machines are the impact or total pressures which take such velocities into account so that (v) and (v + dv) may be considered zero. In other words, the compressor is given the benefit of having a 100% efficient nozzle on the inlet side and a 100% efficient diffuser on the exhaust side. These reduce the

general energy equation to:

$$dL = C_v dT + \frac{1}{J} PdV + \frac{1}{J} VdP \tag{2}$$

or since

$$C_v dT + \frac{1}{J} PdV + \frac{1}{J} VdP = C_v dT + \frac{1}{J} d(PV) = C_p dT \tag{3}$$

$$dL = C_p dT \tag{4}$$

For a compression pressure ratio of $\left(\frac{P_2}{P_1}\right)$, the ideal temperature rise would be given as:

$$\left(\frac{T_2^1}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} \tag{5}$$

Where (T_2^1) is the temperature reached by an ideal process, the ideal temperature rise is:

$$T_2^1 - T_1 = T_1\left(\frac{T_2^1}{T_1} - 1\right) = T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] \tag{6}$$

The adiabatic compression efficiency $\left[N_c\right]$ is defined as

$$N_c = \frac{T_2^1 - T_1}{T_2 - T_1} \tag{7}$$

From Equations (6) and (7), the actual temperature rise is

$$T_2 - T_1 = \frac{T_2^1 - T_1}{N_c} = \frac{T_1}{N_c} \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] \tag{8}$$

The actual compression work from Equation (4) now becomes

$$dL = C_p dT = C_p(T_2 - T_1) = \frac{C_p T_1}{N_c} \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] \tag{9}$$

Equation (9) states the compression work for a given pressure ratio

$\left(\frac{P_2}{P_1}\right)$. An examination of equation (9) reveals that for a given pressure ratio, the compression work decreases directly with the discharge temperature (T_2) for any given inlet temperature (T_1).

In slow speed reciprocating machines, it is possible to cool the discharge temperature by heat transfer during compression but in the high speed continuous flow machines necessary for an efficient gas turbine cycle, this is impossible. Thus, the idea of evaporating water in the air during compression to hold the temperature down and reduce the compression work has a great appeal. It is the purpose of this paper to investigate the actual increase in efficiency which may be obtained by wet compression and some of the aspects and effects of wet compression.

THEORY OF WET COMPRESSION

In order to make the process of wet compression clear, it is well to follow the process in the compressor. A homogeneous mixture of dry air and moisture enters the compressor. We must assume that at any given point during the compression, enough of the moisture will be evaporated to saturate the dry air at the temperature obtained at that point. Thus, in the compressor, we have a mixture of saturated air and moisture. However, as compression continues, the temperature of the mixture is raised. If only dry air were present, for a given pressure increase (dP) the temperature would rise an amount (dT^1). However, this higher temperature permits the air to absorb more moisture and the temperature is decreased by the heat absorbed by the evaporating moisture and also, to a much smaller extent, by the heat absorbed in raising the

temperature of the yet unevaporated moisture. Thus the actual temperature rise is by an amount (dT) which is less than (dT^1). This process is repeated from point to point or from stage to stage giving a cumulative effect much like compound interest.

The case of reversible compression with no heat transfer is defined by saying that the change in internal energy is caused entirely by the compression work or

$$JC_V dT + P_T dV = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where (P_T) is the total pressure of the working medium.

From the gas equation $P_T V = RT$

$$P_T dV + V dP_T = R dT$$

$$\text{or} \quad P_T dV = R dT - V dP_T = R dT - RT \frac{dP_T}{P_T} \quad \ldots\ldots\ldots\ldots (11)$$

From equations (10) and (11) we obtain

$$JC_V dT + R dT - RT \frac{dP_T}{P_T} = 0$$

$$JC_p \frac{dT}{T} = R \frac{dP_T}{P_T}$$

$$\frac{dT}{T} = \frac{R}{JC_p} \frac{dP_T}{P_T} = \frac{\gamma - 1}{\gamma} \frac{dP_T}{P_T}$$

or since we have now a reversible process

$$\frac{dT^1}{T_1} = \frac{\gamma - 1}{\gamma} \frac{dP_T}{P_{T_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Equation (12) expresses the ideal temperature rise (dT^1) for a given pressure rise (dP_T) for a reversible process. Since we are investigating a non-reversible compression, the actual temperature rise (dT)

for a given pressure rise (dP) will be greater than the ideal temperature rise (dT^1) or $dT > dT^1$.

From equation (7)

$$N_c(T-T_1) = T^1-T_1$$

$$N_c dT = dT^1$$

$$N_c\frac{dT}{T_1} = \frac{dT^1}{T_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Substituting $N_c\frac{dT}{T_1}$ for $\frac{dT^1}{T_1}$ in equation (12) gives the actual temperature rise as

$$\frac{dT}{T_1} = \frac{1}{N_c}\,\frac{\gamma-1}{\gamma}\,\frac{dP_T}{P_{T_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Since the actual process is polytropic, the polytropic ratio of specific heats (k) must be substituted for the adiabatic ratio (γ) and equation (14) becomes

$$\frac{dT}{T_1} = \frac{1}{N_c}\,\frac{k-1}{k}\,\frac{dP_T}{P_{T_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

(k) is the ratio of specific heat at constant pressure divided by the specific heat at constant volume for a polytropic process. Equation (15) expresses the actual temperature rise (dT) obtained during wet compression for a given change in total pressure (dP_T).

If (W_w) pounds of moisture are added per pound of entering air, at any given point during the compression process, an amount (W_v) has evaporated to saturate the air. This amount of evaporated moisture will occupy the same volume (V_v) as the air, allowing us to express the specific volume of the air as

$$V_v = W_v V_g \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where (Vg) is the specific volume of the evaporated moisture at the temperature obtained at the given point.

The volume occupied by the saturated air (V_v) will be less than the volume (V) which would be occupied if all the moisture were evaporated by an amount (V_w) occupied by the unevaporated moisture. The volume (V_w) may be evaluated as

$$V_w = \frac{\left(W_w - W_v\right)RT}{P_T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

From equations (16) and (17)

$$Vv = W_v Vg = V - \frac{\left(W_w - W_v\right)RT}{P_T}$$

from which

$$V = W_v V_g + \frac{\left(W_w - W_v\right)RT}{P_T} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

The ratio of the partial pressure of the vapor (p) and the partial pressure of the dry air (P) is in the ratio of the moles of vapor to the moles of dry air, so using the molecular weight of dry air as 29, and that of the vapor as 18,

$$\frac{\frac{1}{29}\text{ moles of air}}{\frac{W_v}{18}\text{ moles of vapor}} = \frac{P}{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

from which, we obtain as an expression for (Wv)

$$Wv = \frac{18}{29}\,\frac{p}{P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

Expanding equation (18) and collecting terms

$$V = W_v V_g + \frac{W_{w_1} RT}{P_T} - \frac{W_v RT}{P_T} = W_v\left(V_g - \frac{RT}{P_T}\right) + \frac{W_{w_1} RT}{P_T} \quad \ldots\ldots (20)$$

Substituting for W_v

$$V = \frac{18}{29}\frac{p}{P}\left(V_g - \frac{RT}{P_T}\right) + \frac{W_w RT}{P_T} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

For a pound of dry air, the entropy change of a reversible process is

$$ds = \frac{dQ}{T_1} = \frac{C_v d\,T + P_T dV}{T_1} = C_v \frac{dT}{T_1} + \frac{R}{J}\frac{dV}{V_1} \quad \ldots\ldots\ldots (22)$$

(V) is the specific volume of dry air or the volume occupied by a pound of dry air. The moisture and the dry air occupy the same volume and (V) is also the volume occupied by the saturated air. Since equation (21) corrects this volume for the amount of unevaporated moisture present, we substitute equation (21) for (V) in equation (22).

From equations (15), (21) and (22)

$$dS = \frac{1}{N_c} C_v \frac{k-1}{k}\frac{dP_T}{P_{T_1}} + \frac{R}{J}\frac{dV}{V}$$

or

$$\triangle S = \frac{1}{N_c} C_v \frac{k-1}{k} \ln \frac{P_T}{P_{T_1}} + \frac{R}{J} \ln \frac{V}{V_1} \quad \ldots\ldots\ldots\ldots\ldots (23)$$

Substituting equation (22) for (V) we obtain

$$\triangle S = \frac{C_v}{N_c}\frac{k-1}{k} \ln \frac{P_T}{P_{T_1}} + \frac{R}{J} \ln \left[\frac{\frac{18}{29}\frac{p}{P}\left(V_g - \frac{RT}{P_T}\right) + \frac{W_w RT}{P_T}}{\frac{18}{29}\frac{P_1}{P_1}\left(V_{g_1} - \frac{RT_1}{P_{T_1}}\right) + \frac{W_w RT_1}{P_{T_1}}}\right] \quad \ldots (24)$$

For a polytropic process

$$\frac{T}{T_1} = \left(\frac{P_T}{P_{T_1}}\right)^{\frac{k-1}{k}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

or
$$\frac{k-1}{k} \log_e \frac{P_T}{P_{T_1}} = \log_e \frac{T}{T_1}$$

from which

$$k - 1 = k \left[\frac{\log_e T - \log_e T_1}{\log_e P_T - \log_e P_{T_1}}\right]$$

$$k - k \left[\frac{\log_e \frac{T}{T_1}}{\log_e \frac{P_T}{P_{T_1}}}\right] = 1$$

$$k \left\{1 - \frac{\log_e \frac{T}{T_1}}{\log_e \frac{P_T}{P_{T_1}}}\right\} = 1$$

$$k = 1 - \frac{1}{\frac{\log_e \frac{T}{T_1}}{\log_e \frac{P_T}{P_{T_1}}}} = \frac{\log_e \frac{P_T}{P_{T_1}}}{\log_e \frac{P_T}{P_{T_1}} - \log_e \frac{T}{T_1}} \quad \ldots\ldots\ldots\ldots \quad (26)$$

Since
$$k = \frac{C_p}{C_v} = \frac{C_v + \frac{R}{J}}{C_v}$$

$$kC_v = C_v + \frac{R}{J}$$

$$C_v(k-1) = \frac{R}{J}$$

$$C_v = \frac{R}{J} \frac{1}{k-1}$$

and $$C_v \frac{k-1}{k} = \left(\frac{R}{J}\right)\left(\frac{1}{k-1}\right)\left(\frac{k-1}{k}\right) = \frac{R}{Jk} \qquad (27)$$

Equation (27) may be written from equation (26) as

$$C_v \frac{k-1}{k} = \frac{R\left(\log_e \frac{P_T}{P_{T_1}} - \log_e \frac{T}{T_1}\right)}{J \log_e \frac{P_T}{P_{T_1}}} \qquad (28)$$

Substituting equation (28) in equation (24) gives

$$\Delta S = \frac{R\left(\log_e \frac{P_T}{P_{T_1}} - \log_e \frac{T}{T_1}\right)\log_e\left(\frac{P_T}{P_{T_1}}\right)}{J N_c \log_e\left(\frac{P_T}{P_{T_1}}\right)} + \frac{R}{J}\log_e\left[\frac{\frac{18}{29}\frac{p}{P}\left(V_g - \frac{RT}{P_T}\right) + \frac{W_w RT}{P_T}}{\frac{18}{29}\frac{p_1}{P_1}\left(V_{g_1} - \frac{RT_1}{P_{T_1}}\right) + \frac{W_w RT_1}{P_{T_1}}}\right]$$

$$\text{or } \Delta S = \frac{R}{JN_c}\left(\log_e \frac{P_T}{P_{T_1}} - \log_e \frac{T}{T_1}\right) + \frac{R}{J}\log_e\left[\frac{\frac{18}{29}\frac{p}{P}\left(V_g - \frac{RT}{P_T}\right) + \frac{W_w RT}{P_T}}{\frac{18}{29}\frac{p_1}{P_1}\left(V_{g_1} - \frac{RT_1}{P_T}\right) + \frac{W_w RT_1}{P_{T_1}}}\right] \qquad (29)$$

The amount of heat (ΔQ) lost during the compression will, at any given instant, be the sum of the heat lost in evaporating (W_v) pounds of moisture and the heat lost in heating the ($W_w - W_v$) pounds of unevaporated moisture to the temperature (T) attained at that point. Letting (h_{f_g}) be the mean heat of evaporation evaluated at the average temperature of the compression up to the given point,

$$\Delta Q = -\left[W_v h_{f_g} + (W_w - W_v)(h_w - h_{w_1})\right] \qquad (30)$$

then $$\Delta S = \frac{\Delta Q}{T_{av}} = -\frac{\left[W_v h_{f_g} + (W_w - W_v)(h_w - h_{w_1})\right]}{T_{av}} \qquad (31)$$

Since the heat is removed over a range of temperature, equation (31) will only be a close approximation of the entropy change, obtained

by using the average temperature instead of a differential form.

From equations (29) and (31) is obtained

$$-\frac{18}{29}\frac{p}{P}h_{fg} - \left(W_w - \frac{18}{29}\frac{p}{P}\right)(h_w - h_{w1}) = \frac{nT_{av}}{JM_c}\left(\log_e \frac{P_T}{P_{T_1}} - \log_e \frac{T}{T_1}\right) +$$

$$\frac{R}{J}\log_e \left[\frac{\frac{18}{29}\frac{p}{P}\left(V_g - \frac{RT}{P_T}\right) + \frac{W_w RT}{P_T}}{\frac{18}{29}\frac{p_1}{P_1}\left(V_{g1} - \frac{RT_1}{P_{T_1}}\right) + \frac{W_w RT_1}{P_{T_1}}}\right] \quad . \; . \; . \; . \; (32)$$

Equation (32) will give the actual pressure (P_T) obtained by wet compression over a temperature range from an initial temperature (T_1) to a discharge temperature (T) for a given amount of moisture (W_w) injected into the compressor inlet air. Equation (32) does not admit to an easy solution for P_T but is not too difficult to solve by a "trial-and-error" method.

The actual compression work is given by equation (2) as

$$dL = C_v dT + \frac{1}{J}\, d(PV) = dh_c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (33)$$

where dh_c is the total change in enthalpy for both the air and the vapor. Considering the enthalpy change of a pound of dry air and (W_v) pounds of vapor per pound of dry air

$$\text{Compression work} = \int dL = \int C_p dT + \int d(W_v h_g)$$

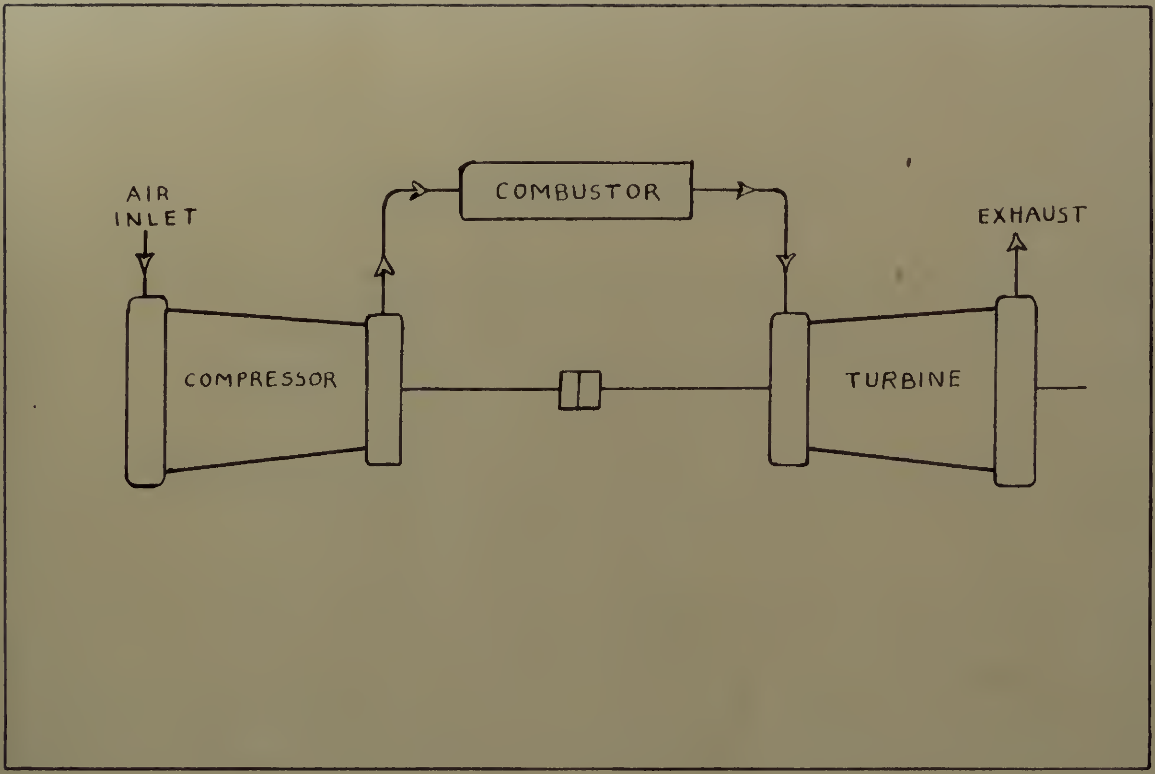
or

$$\text{Compression work} = C_p\,(T - T_1) + W_v h_g - W_{v_1} h_{g_1} \quad . \; . \; . \; . \; . \; (34)$$

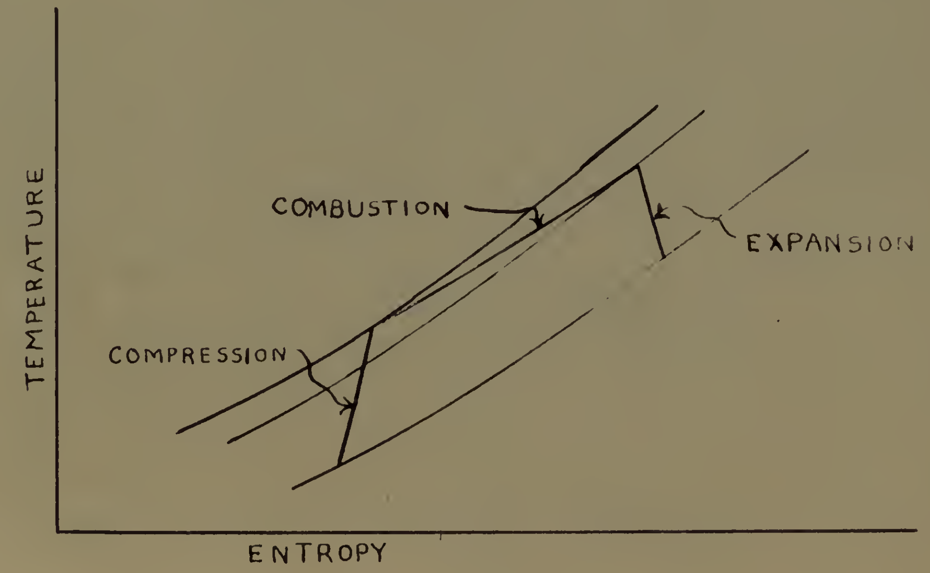
Equation (34) implies that during wet compression the air follows a polytropic compression line, as does equation (32). In actual practice this is probably not strictly true. Although the end points of the compression may lie on a polytropic line, points in between may depart from the polytropic process. Professor Neil P. Bailey of Rensselaer Polytechnic

Institute has shown that if the actual compression work is expressed as $dL = \frac{1}{J} V dP_T$; where P_T is the total pressure, the actual work is $dL = \left[C_p dT + d(W_v h_g)\right] - \left[C_v dT + d(W_v u) + \frac{P_T}{J} dV\right]$. The second term of this equation becomes zero if the change of internal energy of the mixture is equal to the change of internal energy of the vapor, a condition which is true only if the mixture is in equilibrium at all times during the process. In other words, equation (34) is true only if we assume that the air remains saturated throughout the compression. Since in modern high speed compression, the air and moisture remain in the compressor for a very short time, it is probable that equilibrium is not attained until after compression and thus the actual work computed by equation (34) will be somewhat high. Further data on wet compression is necessary to determine the magnitude of the error.

A prediction of the exact effect of a change of compressor efficiency on the overall gas turbine cycle efficiency would require calculation of considerable complexity, so that a rough approximation of the effect is highly desirable. If we take as our cycle the basic 1-0-0 cycle [1 turbine and burner, no compressor intercooler, no regenerator] such as sketched in Figure I we can make a rough approximation without involving errors of too great magnitude. A similar method can be used for any cycle.



SCHEMATIC DIAGRAM OF 1-0-0 GAS TURBINE PLANT.



T.-S. DIAGRAM FOR 1-0-0 GAS TURBINE PLANT.

FIGURE I

The overall cycle efficiency (N) is given by the ratio of the net work output divided by the heat input.

$$N = \frac{\text{turbine work - compressor work - losses}}{\text{heat input}}$$

$$N = \frac{N_T \Delta h_T - \frac{1}{N_c}\Delta h_c - \Delta h_L}{\frac{1}{N_B}\Delta h_B} \quad \dots\dots\dots\dots\dots\dots\dots (35)$$

Where the subscript (T) denotes turbine

(C) denotes compressor

(L) denotes losses

(B) denotes burner.

Assume that the compressor efficiency is changed by a fraction $\frac{\Delta N_c}{N_c} = \varepsilon_c$. The heat increase ($\Delta h_c$) in the compressor will change to $\frac{\Delta h_c}{1+\varepsilon_c}$.

The new efficiency (N^1) now becomes

$$N^1 = \frac{N_T \Delta h_T - \frac{\Delta h_c}{N_c(1+\varepsilon_c)} - \Delta h_L}{\frac{1}{N_B}\Delta h_B} \quad \dots\dots\dots\dots\dots\dots\dots (36)$$

By adding and subtracting $\frac{1}{N_c}\Delta h_c$ to the numerator of equation (36) we can write

$$N^1 = \frac{N_T \Delta h_T - \frac{1}{N_c}\Delta h_c - \Delta h_L + \frac{1}{N_c}\Delta h_c - \frac{\Delta h_c}{N_c(1+\varepsilon_c)}}{\frac{1}{N_B}\Delta h_B}$$

$$N^1 = \frac{N_T \Delta h_T - \frac{1}{N_c}\Delta h_c - \Delta h_L}{\frac{1}{N_B}\Delta h_B} + \frac{\frac{1}{N_c}\Delta h_c - \frac{\Delta h_c}{N_c(1+\varepsilon_c)}}{\frac{1}{N_B}\Delta h_B} \quad \dots\dots\dots (37)$$

From equations (37) and (35)

$$N^1 = N + \frac{\frac{1}{N_c}\Delta h_c - \frac{\Delta h_c}{N_c(1+\mathcal{E}_c)}}{\frac{1}{N_B}\Delta h_B}$$

or

$$N^1 = N + \frac{\frac{\Delta h_c(1+\mathcal{E}_c) - \Delta h_c}{N_c(1+\mathcal{E}_c)}}{\frac{1}{N_B}\Delta h_B} = N + \frac{\frac{\Delta h_c(1+\mathcal{E}_c - 1)}{N_c(1+\mathcal{E}_c)}}{\frac{1}{N_B}\Delta h_B}$$

$$N^1 = N + \frac{(\Delta h_c)(\mathcal{E}_c)(N_B)}{(\Delta h_B)(1+\mathcal{E}_c)(N_c)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

Equation (38) provides a means of roughly evaluating the effect of a change of compressor efficiency on the overall cycle efficiency. It is not accurate, in that it does not include the effect of the change of cycle efficiency on the turbine work and the heat input but it will be an under prediction rather than over prediction in the case of a compressor cycle efficiency increase and will therefore be on the "safe side."

SAMPLE CALCULATIONS

For saturated air at 80° F at a total initial pressure of 14.7 $^{lbs}/_{in^2}$abs

$T_1 = 540°R$

$P_{T_1} = 14.7\ ^{lbs}/_{in^2}$

$P_1 = .5067\ ^{lbs}/_{in^2}$ (from steam tables)

$P_1 = 14.7 - .5067 = 14.1933\ ^{lbs}/_{in^2}$

$h_{w_1} = 48\ \frac{BTU}{lb}$ (from steam tables)

$V_{g_1} = 633.8\ \frac{ft^3}{lb}$ (from steam tables)

Assuming a polytropic compression which gives a final temperature of 200° F with 0.1 pound of moisture added per pound of dry air at the inlet

$T = 660^\circ R$

$p = 11.53 \ \mathrm{lbs/in^2}$ (from steam tables)

$P = p_T - 11.53 \ \mathrm{lbs/in^2}$

$V_g = 33.64 \ \frac{ft^3}{lb}$ (from steam tables)

$$\text{mean } h_{f_g} = \frac{h_{f_g}^{200} + h_{f_g}^{80}}{2} = \frac{979 + 1049}{2} = 1014 \ \frac{BTU}{lb}$$ (from steam tables)

$h_w = 168 \ \frac{BTU}{lb}$

$W_{w_1} = .1$ lb of moisture/lb dry air

$T_{av} = 600^\circ R$

Assuming the dry efficiency of the compressor to be .8 we can break equation (32) into components for ease in calculation and obtain the following:

$$\left[\text{FIRST TERM}\right] - \left[\frac{18}{29}\frac{p}{P}h_{f_g}\right] = -\left[\left(\frac{18}{29}\right)\left(\frac{11.53}{P_T - 11.53}\right)(1014)\right] = -\frac{7,260}{P_T - 11.53}$$

$$\left[\text{SECOND TERM}\right] - \left[\left(W_w - \frac{18}{29}\frac{p}{P}\right)(h_w - h_{w_1})\right] = -\left[\left(0.1 - \left(\frac{18}{29}\right)\left(\frac{11.53}{P_T - 11.53}\right)\right)\right]\left[168 - 48\right]$$

$$= -\left[12 - \frac{860}{P_T - 11.53}\right] = \frac{860}{P_T - 11.53} - 12$$

$$\left[\text{THIRD TERM}\right] \frac{RT_{av}}{J\eta_c}\left(\log_e \frac{P_T}{P_{T_1}} - \log_e \frac{T}{T_1}\right) = \frac{(53.3)(600)}{(778)(.8)}\left(\log_e \frac{P_T}{14.7} - \log_e \frac{660}{540}\right)$$

$$= (51.3)\left(\log_e \frac{P_T}{14.7} - .2\right)$$

$$\left[\text{FOURTH TERM}\right] \frac{RT_{av}}{J}\log_e\left[\frac{\frac{18}{29}\frac{p}{P}\left(V_g - \frac{RT}{P_T}\right) + \frac{W_w RT}{P_T}}{\frac{18}{29}\frac{p_1}{P_1}\left(V_{g_1} - \frac{RT_1}{P_{T_1}}\right) + \frac{W_w RT_1}{P_{T_1}}}\right] =$$

$$\frac{(53.3)(600)}{778}\log_e\left[\frac{\left(\frac{18}{29}\right)\left(\frac{11.53}{P_T - 11.53}\right)\left(33.64 - \frac{(53.3)(660)}{P_T}\right) + \frac{(.1)(53.3)(660)}{P_T}}{\frac{18}{29}\left(\frac{.5067}{14.7}\right)\left(633.1 - \frac{(53.3)(540)}{14.7}\right) + \frac{(.1)(53.3)(540)}{14.7}}\right]$$

$$41.1\log_e\left[\frac{\left(\frac{7.15}{P_T - 11.53}\right)\left(33.64 - \frac{35,200}{P_T}\right) + \frac{3,520}{P_T}}{106.6}\right]$$

The actual solution is obtained with the following tabular form:

Assumed

(1)	P_T	90	80	78	79	79.5
(2)	1st term	-92.6	-106.2	-109.1	-107.7	-106.5
(3)	2nd term	-1.05	0.55	.95	.75	.85
(4)	(2) + (3)	-93.65	-105.65	-108.15	-106.95	-105.65
(5)	3rd term	82.6	76.6	75.4	76.0	76.4

(6)	4th term	-133.8	-175.0	-275.0	-213.8	-200.5
(7)	(5) + (6)	-51.2	-98.4	-199.6	-137.8	-124.1
(8)	(4) - (7)	-42.45	-7.25	91.45	30.85	18.45

The correct solution of (P_T) is very close to 80 pounds per square inch. This means that with wet compression and a non-reversible cycle between 80° F and 200° F, the partial pressure of the air goes from 14.7 lbs./in² to 68.5 lbs/in². The dry air is compressed through a ratio of $\frac{68.5}{14.7} = 4.66$ and the total mixture is compressed through a ratio of $\frac{80}{14.7} = 5.44$.

CALCULATION OF IDEAL PRESSURE RATIO

For ideal wet compression, Prof. Neil P. Bailey developed the equation

$$C_v \log_e \frac{T}{T_1} + \frac{R}{J} \log_e \left[\frac{\frac{pV_g}{P}}{\frac{p_1V_{g1}}{P_1}} \right] + \frac{18}{29} \left[\frac{pS_g}{P} - \frac{p_1S_{g1}}{P_1} \right] = 0$$

in his book on Thermodynamic Cycles.

For 80° F saturated air at a total initial pressure of 14.7 lbs. per sq. in. absolute,

$T_1 = 540°$ R, $p_1 = .5069$ lb. per sq. in.

$P_1 = 14.7 - .5069 = 14.193$ lbs. per sq. in.

From steam tables $V_{g1} = 633.1$ cu. ft. per lb.

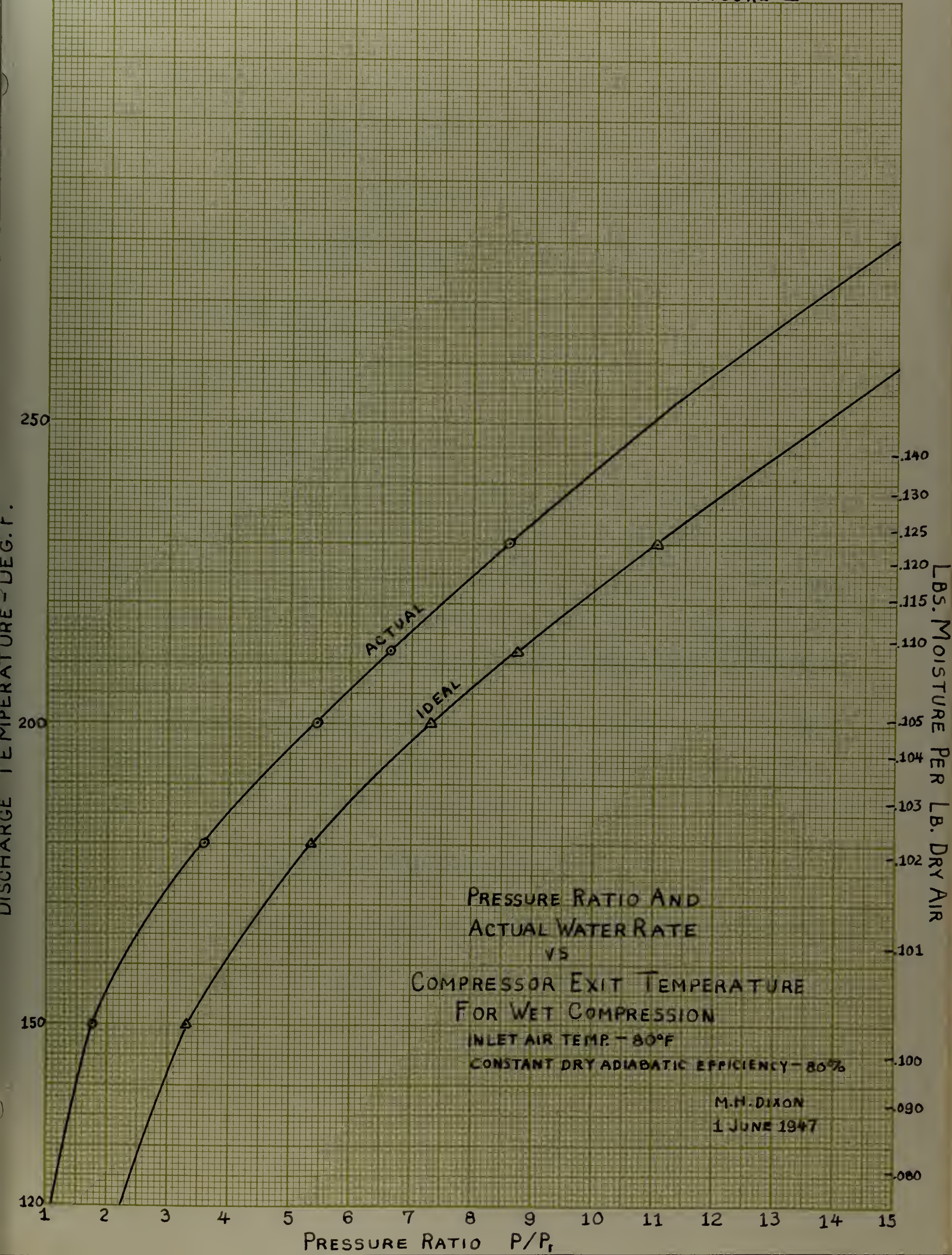
Assuming the moisture to be supplied at 80° F, $S_{g1} = 1.9428$.

Now assuming a reversible compression that gives a final temperature of 150° F, $T = 610°$ R; $p = 3.718$ lbs. per sq. in.

$V_g = 97.07$ cu. ft. per lb.

$S_g = S_g^{150} - S_g^{80} = 1.8685 - .0932 = 1.7753$

FIGURE II



PRESSURE RATIO AND
ACTUAL WATER RATE
VS
COMPRESSOR EXIT TEMPERATURE
FOR WET COMPRESSION
INLET AIR TEMP. - 80°F
CONSTANT DRY ADIABATIC EFFICIENCY - 80%

M.H. DIXON
1 JUNE 1947

Using $C_v = .172$

$$.172 \log_e \frac{610}{540} + \frac{53.3}{778} \log_e \frac{(3.718)(97.07)}{(.5069)(633.1)} - \frac{53.3}{778} \log_e \frac{P}{14.193} +$$

$$\frac{18}{29}\left[\frac{(3.718)(1.7753)}{P}\right] - \frac{18}{29}\left[\frac{(.5069)(1.9428)}{14.193}\right] = 0$$

$$.0224 + .0064 - .0685 \log_e \frac{P}{14.193} + \frac{4.1}{P} - .0432 = 0$$

or $$.0685 \log_e \frac{P}{14.193} - \frac{4.1}{P} = -.0124$$

For $P = 45$ lbs. per sq. in.

$$.0792 - .0912 = -.0124$$

$$-.0120 \approx -.0124$$

This means that with wet compression on a reversible cycle between 80° F and 150° F, the partial pressure of the air goes from 14.193 to 45 lbs. per sq. in. and the saturated vapor goes from .5069 to 14.7 lbs. per sq. in. The dry air is compressed through a ratio of $\frac{45}{14.193} = 3.17$ and the total mixture is compressed through a ratio of $\frac{45 + 3.718}{14.7} = 3.31$.

Figure II shows curves of actual and ideal pressure ratios obtained for various discharge temperatures and the pounds of moisture per pound of air added for actual wet compression.

INCREASE IN COMPRESSOR EFFICIENCY

The amount of moisture (W_v) evaporated during the compression would be

$W_v = \frac{18}{29}\frac{p}{P} = \frac{(18)}{29}\frac{(11.53)}{68.5} = .1045$ lbs. moisture/lb. air.

Ideal wet compression work $= W_{c_i} = C_p T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{R}{JC_{pa}}} - 1\right] =$

$$\frac{k}{k-1}\frac{R}{J} T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

$$T_1 = 540^\circ\ R,\ T_2 = 660^\circ\ R,\ \frac{P_2}{P_1} = 5.44,\ w_w = .1045$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{k-1}{k} \log_e \frac{P_2}{P_1} = \log_e \frac{T_2}{T_1}$$

$$\frac{k-1}{k} = \frac{\log_e \frac{T_2}{T_1}}{\log_e \frac{P_2}{P_1}}$$

$$\frac{k-1}{k} = \frac{\log_e \frac{660}{540}}{\log_e 5.44} = \frac{.2007}{1.6938} = .1182$$

Gas constant of mixture $= R_m = \frac{(85.6)(.1045)}{1.1045} + \frac{(1)(53.3)}{1.105} = 56.33$

$$E_{c_1} = \frac{(56.33)(540)}{(.1182)(778)} \left[(5.44)^{.1182} - 1\right] = 330 \left[.222\right] = 73.4 \frac{\text{BTU}}{\text{lb. mixture}}$$

To accomplish the same result with a reversible dry compression would result in a final temperature of

$$T^1 = (540)(5.44)^{\frac{.395}{1.395}} = 872^\circ\ R$$

and would require $C_p(T^1 - T_1) \frac{\text{BTU}}{\text{lb. air}}$ or

$$E_{cd_1} = (.240)(872 - 540) = 79.7\ ^{\text{BTU}}/\text{lb air}$$

The ideal wet compression work per pound of air is

$$73.4 \frac{\text{BTU}}{\text{lb mixture}} \times \frac{1.1045 \text{ lb mixture}}{1 \text{ lb air}} = 81.2 \frac{\text{BTU}}{\text{lb air}}$$

Assuming a dry compression $N_{cd} = .8$, the actual dry compression work per pound of air is

$$\frac{E_{cd}(\text{ideal})}{N_{cd}} = \frac{79.7}{.8} = 99.6 \frac{\text{BTU}}{\text{lb air}}$$

The actual wet compression work is given by equation (34) as actual compression work $= Cp(T - T_1) + W_v h_g - W_{v_1} h_{g_1}$

$$W_{v_1} = \frac{RT_1}{P_1 V_1} = \frac{(53.3)(540)}{(14.193)(144)(633.1)} = .0223 \text{ lbs vapor/lb air}$$

$$\text{Actual compression work} = .240\ (200 - 80) + (.1045)(978) - (.0223)(1048)$$

$$= 28.8 + 102.2 - 23.37$$

$$\text{Actual compression work} = 107.63\ \frac{\text{BTU}}{1.105 \text{ lb mixture}} = 107.63\ \frac{\text{BTU}}{\text{lb air}}$$

The actual compression work per pound of mixture is

$$107.63\ \frac{\text{BTU}}{\text{lb air}} \times \frac{1 \text{ lb air}}{1.105 \text{ lb mixture}} = 97.3\ \frac{\text{BTU}}{\text{lb mixture}}$$

Table I lists the results computed this far for comparison.

Table I

Ideal dry compression work	$79.7\ \frac{\text{BTU}}{\text{lb air}}$	
Actual dry compression work with $N_{c_d} = .8$	$99.6\ \frac{\text{BTU}}{\text{lb air}}$	
Ideal wet compression work	$81.2\ \frac{\text{BTU}}{\text{lb air}}$	$= 73.4\ \frac{\text{BTU}}{\text{lb mixture}}$
Actual wet compression work	$107.63\ \frac{\text{BTU}}{\text{lb air}}$	$= 97.3\ \frac{\text{BTU}}{\text{lb mixture}}$

Since our compressor is a part of a gas turbine cycle, with wet compression the entire 1.1045 pounds of mixture would be useful at the compression ratio of 5.44. Therefore, to accomplish the desired result, the ideal dry compression work would have to be (1.1045)(79.7) = 83.4 $^{\text{BTU}}$ per pound of mixture and the actual compression work (1.1045)(96.6) = 104.2 BTU

per pound of mixture. Table II gives a better comparison of wet and dry compression.

Table II

Ideal dry compression work	83.4 $\frac{BTU}{lb}$
Actual dry compression work with $N_c = .8$	104.2 $\frac{BTU}{lb}$
Ideal wet compression work	73.4 $\frac{BTU}{lb}$
Actual wet compression work	97.3 $\frac{BTU}{lb}$

The adiabatic wet compression efficiency is

$$N_{c_w} = \frac{\text{Ideal dry compression work}}{\text{Actual wet compression work}} = \frac{83.4}{97.3} = .868$$

Thus, wet compression has increased the compressor efficiency from an adiabatic dry compression efficiency of .8 to an adiabatic wet compression efficiency of .868 or an increase of $\frac{.868 - .8}{.8} = .085 = 8.5\%$.

EFFECT ON OVERALL EFFICIENCY

Assuming a fuel with a lower heating value of 19,000 $^{BTU}/lb$ and a fuel air ratio of one pound of fuel to 80 pounds of air

$$\Delta h_B = \frac{19,000}{80} = 237.5 \frac{BTU}{lb\ air}$$

Using $N_B = .98$

$$N_c = .8$$

$$E_c = \frac{\Delta N_c}{N_c} = \frac{.068}{.8} = .085$$

$$\Delta h_c = C_p(T - T_1) = 28.8$$

Equation (38) gives the new overall efficiency N^1 as

$$N^1 = N + \frac{(\Delta h_c)(\varepsilon_c)(N_m)}{(\Delta h_B)(1+\varepsilon_c)(N_c)}$$

$$N^1 = N + \frac{(28.8)(.085)(.98)}{(237.5)(1.085)(.8)} = N + .01162$$

The increase in overall efficiency is .01162. If the overall efficiency (N) of the gas turbine cycle was .25 the new overall efficiency will be .2616 which corresponds to an increase of $\frac{.2616 - .25}{.25} = .0465 = 4.65\%$. Thus an increase in compressor efficiency of 8.5% will increase the overall cycle efficiency by at least 4.65%. As mentioned previously, this figure is somewhat low but it does serve to indicate the value of wet compression as a means of increasing the overall cycle efficiency.

WET COMPRESSION VS. WATER INJECTION

It has long been recognized that two of the main stumbling blocks in the way of gas turbine development are: (1) The small margin between the net work of the turbine and the gross work required by the compressor and auxiliaries, (2) the limiting temperature set by available metals. These may be minimized by the use of water injection. In fact, about thirty years ago, it was suggested by Dr. S. A. Moss and Mr. C. A. Norman of the General Electric Company that water injection in the combustion chamber was a very effective means for lowering the maximum temperature and that it also furnished a working medium which did not have to go through the compressor.

It is easy to deduce that the amount of water necessary for efficient results with water injection, if passed through the compressor, would increase the net compressor work far over the reduction achieved by wet com-

pression with the proper amount of added moisture. Since the water required for water injection is a relatively large amount as compared to the amount for wet compression (up to ten pounds of water per pound of fuel for water injection as compared to around one- to two-tenths pounds of water per pound of fuel for wet compression), it is fairly obvious that the work required to pass this larger amount of water through the compressor would be far from a negligible amount. In addition, there would be the problem of mechanical damage which in all probability would be similar to that caused by the presence of carry-over water in high speed steam turbines.

An analysis of equation (32) also shows that the pressure ratio attained, with large amounts of unevaporated water passing through the compressor, reduces to an undesirably small figure. The second term of the equation reaches a quite large negative value, making the left side of the equation a large negative term. With a given temperature range, the third term becomes somewhat smaller but remains positive until the pressure ratio is reduced to a small figure (less than two). The fourth term, then, must reach a very large negative value to make the two sides of the equation balance. This term attains a large negative value only as the volume occupied by the saturated air and moisture [as expressed by equation (21)] grows larger when compared with the volume occupied by the unevaporated moisture. If we write equation (21) as

$$V = \frac{18}{29}\frac{p}{P}V_g - \frac{1}{P_T}\left(\frac{18}{29}\frac{p}{P}RT - W_wRT\right) \ldots \ldots \ldots \ldots (39)$$

and set V = 0 as a limiting case, the total pressure (P_T) may be expressed as

$$P_T = \frac{\frac{18}{29}\frac{p}{P}RT - W_wRT}{\frac{18}{29}\frac{p}{P}V_g} \ldots \ldots \ldots \ldots (40)$$

From equation (40) it is easily seen that as the amount of water (W_w) is increased the total pressure must decrease in order for the fourth term to attain the large negative value necessary to balance equation (32). This means that the pressure ratio attained for a given temperature range will decrease as the amount of water added is increased over the amount which will just cause saturation of the air at the compressor discharge temperature. It would appear not advisable to attempt water injection by increasing the amount of water, in the compression process, over that necessary for most efficient wet compression.

The best solution to the problem of lowering turbine inlet temperatures while at the same time increasing the net output of the gas turbine plant by reducing the compression work, is a combination of water injection and wet compression with only enough water injected into the compressor inlet for complete evaporation at the discharge temperature. This could easily be achieved by an automatic control such as a thermostatically controlled value regulating the amount of water sprayed into the compressor intake according to the discharge temperature attained.

There are situations, however, where water may not be available for wet compression. The Brown-Boveri Company of Switzerland made a great point of the fact that its first gas turbine plant required no water of any kind. This was due not least to its contemplated use as a reserve plant in case of water supplies disrupted by air raids. Also, although gas turbine design has not yet reached the point where jet engines of the gas turbine type are considered practical for large cargo-passenger planes, it appears distinctly probable that they will be in use in the near future. In both cases, water supplies large enough to maintain at least partial wet compression over a long period of time may be made available to continue.

serve fuel and therefore increase the operating hours attained with a limited fuel supply. Further investigation, of course, would be necessary to determine whether or not, in cases such as that of large aircraft engines, the disadvantages of increased weight and space considerations for water tanks would not overcome the advantage of increased efficiency. But in any case, a small amount could be carried to increase the net work available per pound of fuel during emergency conditions whereas in neither of the two cases cited would it be practical to carry water equal to five to ten times the fuel tank capacity for water injection.

Therefore, wet compression should be definitely advantageous under most conditions and certainly would be highly desirable wherever water injection is possible.

EFFECT OF WET COMPRESSION ON COMPRESSOR STABILITY

A continuous flow compressor with a combustor and fuel system may become unstable and pulsate under certain conditions. Such instability becomes a major operating limitation in gas turbine and jet units. Let us investigate some of the operating principles of an axial-flow air compressor in order to determine the effect of wet compression on pulsation and to find some of the causes of pulsation.

In an axial flow compressor, the air flows essentially in an axial direction. In addition to this motion, the air is given a rotating motion in the direction of rotation of the rotor while passing through the blades. The energy of rotational motion past the rotor blades constitutes a loss which is reduced by the use of stator blades. There is a pressure drop in the stator and a pressure increase in the rotor blades. The total static-pressure increase of the compressor is the algebraic sum of the two pressure changes.

Because of surface friction, eddies, shock, etc., certain losses occur in the flow through the compressor. These losses vary with the deflection through the stator and with the angles of the different blade sections and the magnitude of the relative velocity of flow.

Since the blade velocity at the hub is less than for the rest of the blade, in order to produce the full pressure differential across the wheel by the hub sections, the required blade deflection becomes a maximum at the hub. This leads to large chords (wide blades) and to a high coefficient of lift. For both quantities, however, there exists a definite upper limit. The lift coefficient must be kept low enough so that stalling is avoided. This leads to minimum values for the blade speed at the hub. If the chord is made too long, harmful interference between neighboring blades will result. Tip speeds are limited by the strength of the blades and by the velocity effects as the flow becomes super-acoustic. When this happens, shock waves form and the efficiency drops considerably.

Static pressure is produced by retardation of the velocity relative to the inlet of the blade to a lower relative velocity at the outlet. This is in contra-distinction to the flow in a turbine, in which case, the conversion of pressure into velocity results in an accelerated flow through the blades. In both cases, there is a region of decreased velocity near the surface, caused by surface friction, called the boundary-layer. Whereas in accelerating flow the boundary-layer remains thin because its kinetic energy is replenished from the accelerated stream, in retarding flow it increases rapidly in thickness until the point is reached where separation occurs and further increase of pressure is prevented. The increase of thickness of the boundary-layer in the direction of flow is related to the increase of pressure along the profile of the blade. A part

of this increase of pressure is a consequence of the shape of the blade, particularly in the case of thick blades, since the pressure along the surface decreases first from the impact pressure at the leading edge down to the minimum at the medium curvature of the blade and then gradually rises again toward the trailing edge. The pressure increase is accentuated when the airfoil is placed in a retarding flow.

The increase in boundary layer thickness produces an increase in drag on the blade. It follows from this that conversion of energy in an airfoil grid must take place at a lower efficiency in a retarding flow such as the axial flow compressor, than in an accelerating flow such as the turbine.

The interval between the stator blades of the compressor forms a canal of increasing area in which a pressure increase is produced by diffusion after an initial pressure drop in the rotor blades.

When a decrease of the quantity of air flow through the compressor is enough to cause the lift coefficient to rise beyond a maximum value, stalling occurs. This occurs first at the blade section nearest the hub. As the quantity of flow is decreased below the region of optimum operating conditions, a region of instability results. This comes from the fact that the increasing pressure requires lift forces which are beyond the maximum lift the blade can produce. Consequently, the flow breaks away from the back of the blade and stalling (or flow separation) occurs. This may produce fluctuations of pressure and the quantity of discharge. In the critical range, flow separation alternates with normal flow, producing a peculiar rise and fall of sound.

As the speed of the turbine is reduced, the axial velocity of air flow through the compressor is also decreased. Correspondingly the direction of the air entering the rotor relative to the rotor is changed as

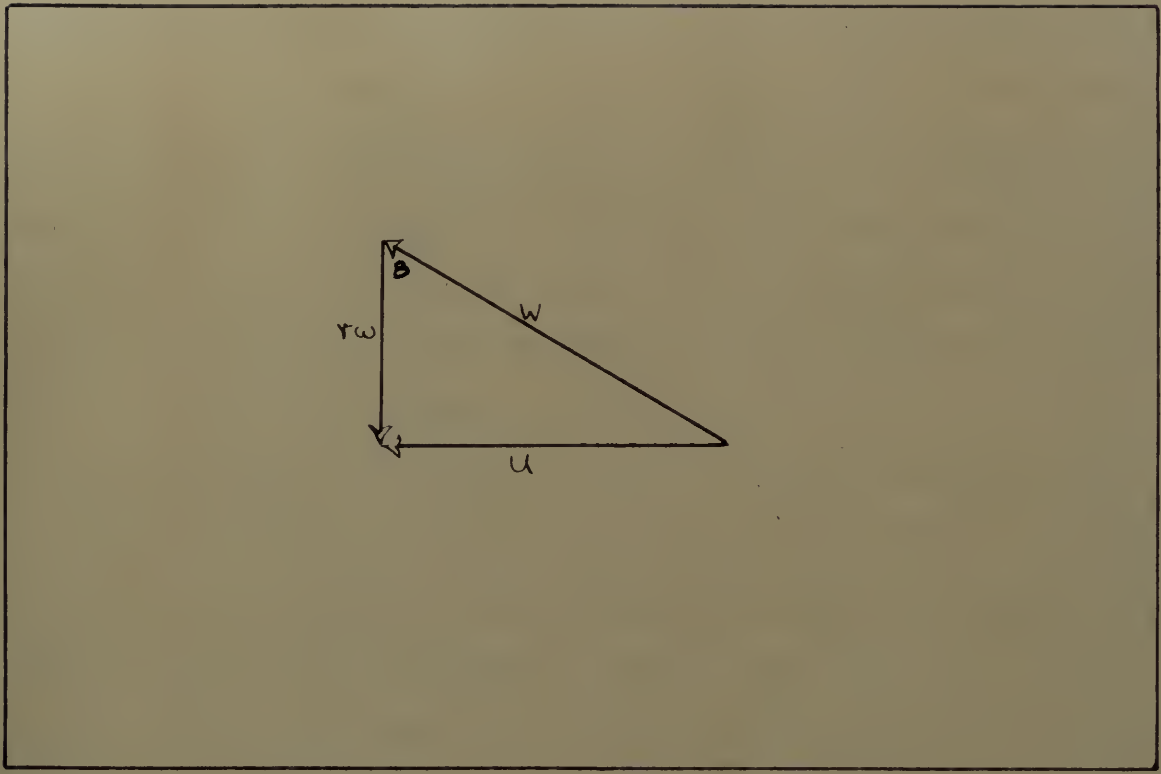


FIG. III VELOCITY DIAGRAM OF AIR ENTERING ROTOR BLADING, SHOWING ROTOR VELOCITY ($r\omega$), AXIAL VELOCITY OF ENTERING AIR (u), VELOCITY OF AIR RELATIVE TO BLADING (w).

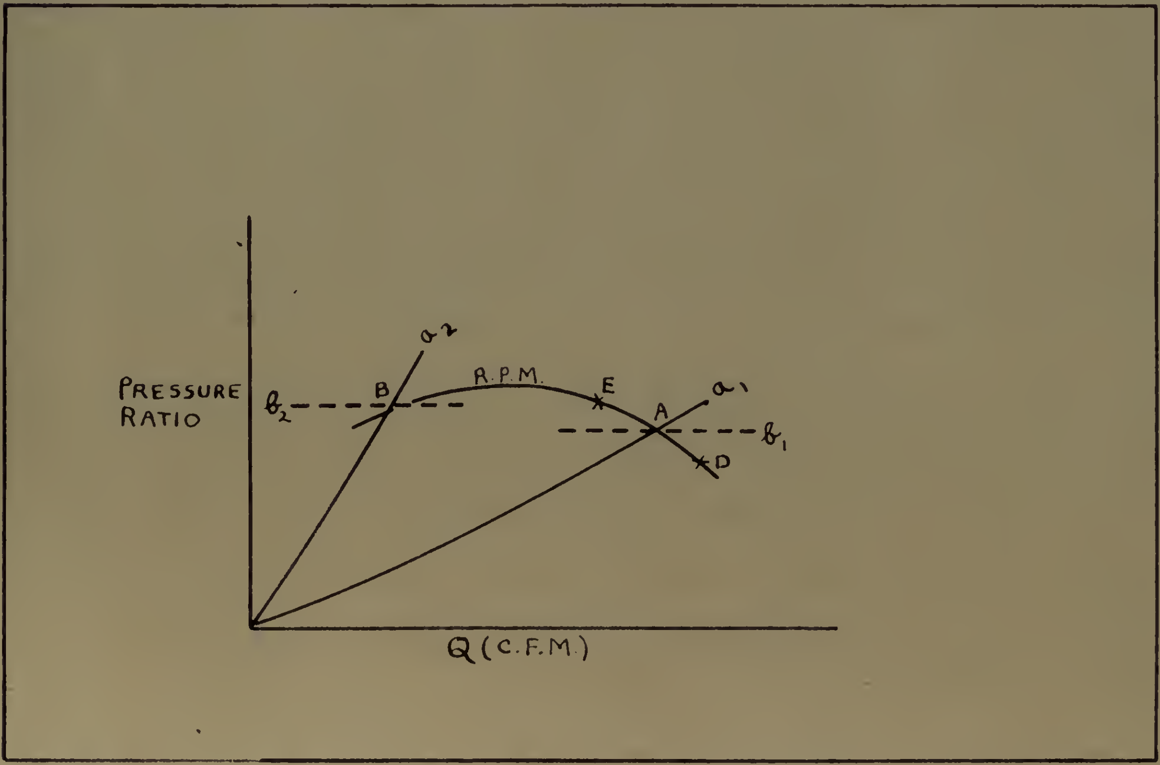


FIG. IV DIAGRAM OF TYPICAL CURVE OF COMPRESSOR R.P.M. VERSUS PRESSURE RATIO AND VOLUMETRIC FLOW.

shown by Fig. III.

As the angle (B) changes, the angle of attack of the rotor blades changes, changing the lift and drag of the blade. As the velocity of flow through the compressor decreases, the angle of attack increases until the stall point for the blade is reached. The phenomenon of flow separation at the lee side of the blade leads to surging. However, separation is not identical with surging.

Take operating point (A) FIG IV lying within the stable operating range. If operating conditions are changed to point (E) where the flow is reduced, the pressure will be higher than the flow resistance and the flow will increase again. If conditions change to point (D), where the flow has increased, the pressure would be lower than flow resistance and the flow would decrease again. The necessary and sufficient conditions for stable operation are given by

$$\left(\frac{dP}{dQ}\right) \text{resistance} > \left(\frac{dP}{dQ}\right) \text{compressor}$$

i.e.: The pressure resistance in the system must increase more rapidly than the compressor pressure during an increase in flow. This condition is also satisfied for point (B). However, the line (a_2) represents a steady state condition and does not apply necessarily to the instantaneous state. With a compressible gas flow, the system has a certain accumulating capacity, depending on the size, as a result of which the pressure of the system, as the flow changes, does not follow curve (a_2) immediately but with a delay in time. With a large system, for a short time following an operating condition change, the system pressure remains constant and may be represented by line (b_2). In this case, the condition of stability is not fulfilled since the supply pressure increases more rapidly with increased flow

than the (momentary) resistance pressure. As a result, the flow at (B) may be unstable and will tend to depart from (E) in either direction. The flow will move back and forth between zero and a point (D) with a period determined by the ratio between the capacity of the system and the quantity being delivered at point (D). Since the separation of flow from the blade is also a function of time, it is possible to have stable operation at point (B). The horizontal line (b_1) at point (A) shows stable operation at all times.

It would therefore seem theoretically possible to eliminate unstable operating ranges of an axial flow compressor by a device which would relieve supply pressure when surging starts.

Professor Neil P. Bailey, Mechanical Engineering Department, Rensselaer Polytechnic Institute, shows the effect of inertia on pulsation in his book "Gas Turbine Combustion and Stability." Take the system shown in Fig. V below, where the net work output of the system is represented by (W_x), the amount of air bled off to make the compressor work equal to the turbine work. Following Professor Bailey's method, we proceed as follows:

From Fig. V during a transient disturbance; in the burner flow

flow rate entering = flow rate leaving + rate of storage

$$W = W_1 - W_x = W_2 + \frac{d\left(\frac{PV}{RT_2}\right)}{dt} \quad . . . . (1)$$

The variation of weight flow through the burner with the pressure (P) [making the assumption that (P) is an average value for the whole system] will be some variation of the curve shown in Fig. VI.

Letting the subscript ($_o$) denote steady state conditions, a disturbance of differential magnitude, or a larger disturbance taken very soon after its creation, may be expressed as:

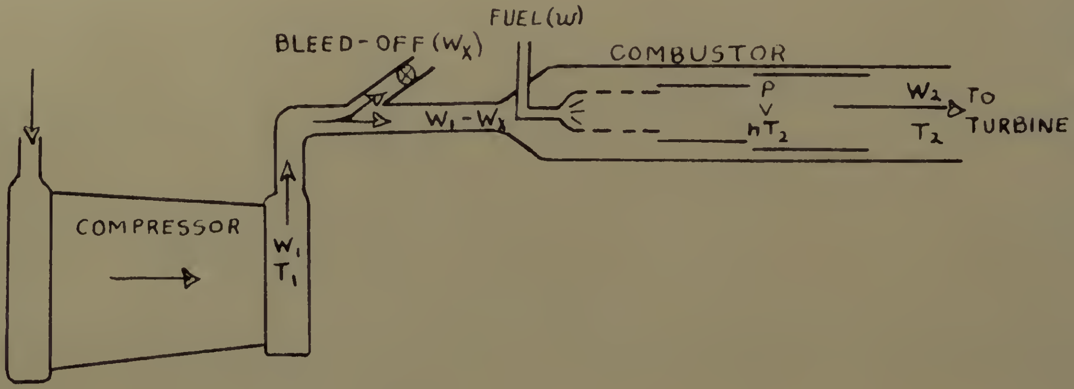


FIG. V SCHEMATIC DIAGRAM OF COMPRESSOR AND BURNER.

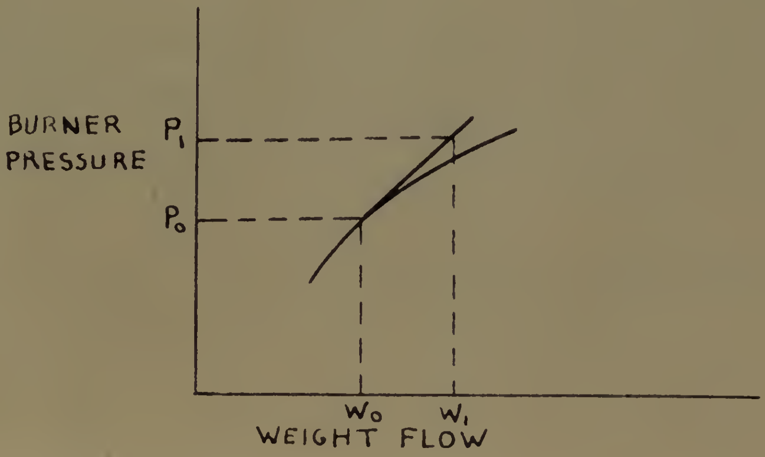


FIG. VI VARIATION OF WEIGHT FLOW WITH PRESSURE.

$$\frac{W_1 - W_{o_1}}{P - P_o} = \frac{dW_1}{dP}$$

or $W_1 = (P - P_o) \dfrac{dW_1}{dP} + W_{o_1}$ . . . . (2)

also $W_2 = (P - P_o) \dfrac{dW_2}{dP} + W_{o_2}$ . . . . (3)

and $W_x = (P - P_o) \dfrac{dW_x}{dP} + W_{o_x}$ . . . . (4)

Where W, W_2, W_x and P are instantaneous conditions. Substituting equations (2), (3) and (4) in (1)

$$(P - P_o) \frac{dW}{dP} + W_o = (P - P_o) \frac{dW_2}{dP} + W_{o_2} + (P - P_o) \frac{dW_x}{dP} + W_{o_x} + \frac{d\left(\frac{PV}{NRT_2}\right)}{dt} \quad ..(5)$$

Since both pressure and temperature vary with time

Rate of storage $= \dfrac{d\left(\frac{PV}{NRT}\right)}{dt} = \dfrac{V}{NRT_2} \dfrac{dP}{dt} - \dfrac{PV}{NRT_2^2} \dfrac{dT_2}{dt}$ . . . . (6)

The usual pressure-weight flow curves for a compressor are taken for steady state conditions and cannot be taken directly to predict transient conditions because they do not take into account inertia effects. One way of including such effects is to say that during a transient, a compressor will deliver a pressure (P) of

$P = P_1 - I \dfrac{dW_1}{dt}$ . . . . (7)

Where (P_1) is the pressure obtained neglecting inertia effects, the rate of pressure change in the system becomes

$\dfrac{dP}{dt} = \dfrac{dP_1}{dt} - I \dfrac{d^2W_1}{dt^2}$ . . . . (8)

Setting up a heat balance across the burner

$(W_1 - W_x)\, C_p\, (T_2 - T_1) = WH$ . . . . (9)

Where H is the heating value of the fuel

differentiating equation (9) with respect to time, assuming

T_1 = constant

$$(W_1 - W_x)\, C_p \frac{dT_2}{dt} + C_p(T_2 - T_1)\left(\frac{dW_1}{dt} - \frac{dW_x}{dt}\right) = H \frac{dw}{dt} \quad \ldots\ldots\ldots\ldots (10)$$

Assuming a high pressure fuel system such that $\frac{dw}{dt} = 0$

$$(W_1 - W_x)\, C_p \frac{dT_2}{dt} = C_p(T_2 - T_1)\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right) \quad \ldots\ldots\ldots\ldots (11)$$

or

$$\frac{dT_2}{dt} = \frac{T_2 - T_1}{W_1 - W_x}\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right)$$

or

$$dT_2 = \frac{T_2 - T_1}{W_1 - W_x}\left(dW_x - dW_1\right) \quad \ldots\ldots\ldots\ldots (12)$$

Substituting equation (11) in (6)

$$\text{rate of storage} = \frac{V}{NRT_2}\frac{dP}{dt} - \frac{PV}{NRT_2^2}\frac{(T_2 - T_1)}{W_1 - W_x}\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right)$$

or

$$\text{rate of storage} = \frac{1}{n}\left[\frac{V}{RT_2}\frac{dP}{dt} - \frac{PV}{RT_2^2}\frac{(T_2 - T_1)}{W_1 - W_x}\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right)\right]$$

or

$$\text{rate of storage} = \frac{V}{NRT_2}\left[\frac{dP}{dt} - \frac{P}{T_2}\frac{(T_2 - T_1)}{W_1 - W_x}\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right)\right] \quad \ldots\ldots\ldots (13)$$

With the burner operating at pressures high enough to have nearly acoustic flow

$$\frac{W_{o2}\sqrt{T_{o2}}}{P_oA} = \text{constant}$$

or since temperatures taken are all total temperatures

$$\frac{W_{o2}\sqrt{T_2}}{P_oA} = \text{constant}$$

or $$W_{o2} = \frac{K P_o}{\sqrt{T_2}} \quad \ldots\ldots (14)$$

Taking log to the base (e)

$$\log_e W_{o2} = \log_e K + \log_e P_o - 1/2 \log_e T_2$$

differentiating

$$\frac{dW_{o2}}{W_{o2}} = \frac{dP_o}{P_o} - \frac{1}{2}\frac{dT_2}{T_2}$$

or $$dW_{o2} = \frac{W_{o2}}{P_o} dP_o - \frac{W_{o2}}{2T_2} dT_2 \quad \ldots\ldots (15)$$

Substituting (12) in (15)

$$dW_{o2} = \frac{W_{o2}}{P_o} dP_o - \frac{W_{o2}}{2T_2} \frac{(T_2 - T_1)}{W_1 - W_x} (dW_x - dW_1) \quad \ldots\ldots (16)$$

At the start of any transient when the changes of (W), (T) and (P) from the steady state values (W_o), (T_o) and (P_o) are small, equation (16) may be integrated to

$$W_2 - W_o = \frac{W_2}{P} (P - P_o) - \left(\frac{W_2}{2T_2}\right) \frac{(T_2 - T_1)}{W_1 - W_x} (W_x - W_{o_x} - W_1 + W_{o_1}) \quad \ldots (17)$$

From equation (7)

$$P - P_o = (P_1 - P_o) - I \frac{dW_1}{dt} \quad \ldots\ldots (18)$$

Also for small changes

$$P_1 - P_o = (W_1 - W_o) \frac{dP_1}{dW_1} \quad \ldots\ldots (19)$$

Substituting (19) in (18)

$$P - P_o = (W_1 - W_o) \frac{dP_1}{dW_1} - I \frac{dW_1}{dt} \quad \ldots\ldots (20)$$

From (20) and (17)

$$W_2 - W_{o_2} = \frac{W_2}{P}\left[(W_1 - W_o)\frac{dP_1}{dW_1} - I\frac{dW_1}{dt}\right] -$$

$$\left(\frac{W_2}{2T_2}\right)\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left[(W_x - W_{o_x}) - (W_1 - W_{o_1})\right] \quad \ldots\ldots (21)$$

During a transient equation (1) becomes

$$(W_1 - W_{o_1}) = (W_2 - W_{o_2}) + (W_x - W_{o_x}) + \text{rate of storage.} \quad \ldots\ldots\ldots (22)$$

Substituting (21) and (13) in (22)

$$W_1 - W_{o_1} = \frac{W_2}{P}\left[(W_1 - W_o)\frac{dP_1}{dW_1} - I\frac{dW_1}{dt}\right] - \left(\frac{W_2}{2T_2}\right)\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left[(W_x - W_{o_x}) - (W_1 - W_{o_1})\right] +$$

$$(W_x - W_{o_x}) + \frac{V}{NRT_2}\left[\frac{dP}{dt} - \frac{P}{T_2}\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right)\right] \quad \ldots\ldots (23)$$

Equation (8) in (23) gives

$$(W_1 - W_{o_1}) = (W_1 - W_{o_1})\left(\frac{W_2}{P}\right)\frac{dP_1}{dW_1} - I\left(\frac{dW_1}{dt}\right)\left(\frac{W_2}{P}\right) - \left(\frac{W_2}{2T_2}\right)\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left[(W_x - W_{o_x}) - (W_1 - W_{o_1})\right]$$

$$+ (W_x - W_{o_x}) + \frac{V}{NRT_2}\left[\left(\frac{dP_1}{dt} - I\frac{d^2W_1}{dt^2}\right) - \frac{P}{T_2}\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right)\right]$$

or since $W_x - W_{o_x} = (P - P_o)\dfrac{dW_x}{dP}$

$$(W_1 - W_{o_1})\left[\frac{W_2}{P}\frac{dP_1}{dW_1} - 1\right] - I\left(\frac{dW_1}{dt}\right)\left(\frac{W_2}{P}\right) - \left(\frac{W_2}{2T_2}\right)\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left[(P - P_o)\frac{dW_x}{dP} - (W_1 - W_{o_1})\right]$$

$$+ (P - P_o)\frac{dW_x}{dP} + \left(\frac{V}{NRT_2}\frac{dP_1}{dW_1}\right)\frac{dW_1}{dt} - \frac{IV}{NRT_2}\frac{d^2W_1}{dt^2} - \frac{PV}{NRT_2^2}\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\left(\frac{dW_x}{dt} - \frac{dW_1}{dt}\right) = 0$$

Substituting $\dfrac{d(W_1 - W_{o_1})}{dt} = \dfrac{dW_1}{dt}$ and $\dfrac{d^2(W_1 - W_o)}{dt^2} = \dfrac{d^2W_1}{dt^2}$ $(W_o = \text{constant})$

$$(W_1 - W_{o_1})\left[\frac{W_2}{P}\frac{dP_1}{dW_1} - 1\right] - \frac{I\,W_2}{P}\frac{d(W_1 - W_{o_1})}{dt} - \left(\frac{T_2 - T_1}{2T_2}\right)\left(\frac{W_2}{W_1 - W_x}\right)\left(W_x - W_{o_x}\right) +$$

$$\frac{(T_2 - T_1)}{2T}\left(\frac{W_2}{W_1 - W_x}\right)(W_1 - W_{o_1}) + (W_x - W_{o_x}) + \left(\frac{V}{NRT_2}\right)\left(\frac{dP_1}{dW_1}\right)\frac{d(W_1 - W_{o_1})}{dt} -$$

$$\frac{IV}{NRT_2}\frac{d^2(W_1 - W_o)}{dt^2} - \frac{PV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x}\frac{dW_x}{dt} + \frac{PV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x}\frac{d(W_1 - W_{o_1})}{dt^2} = 0$$

Collecting like terms

$$\frac{d^2(W_1 - W_{o_1})}{dt^2}\left(-\frac{IV}{NRT_2}\right) + \left[\left(\frac{V}{NRT_2}\right)\left(\frac{dP_1}{dW_1}\right) + \frac{PV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x} - \frac{IW_2}{P}\right]\frac{d(W_1 - W_{o_1})}{dt} +$$

$$\left[\frac{W_2}{P}\frac{dP_1}{dW_1} - 1 + \frac{(T_2 - T_1)}{2T_2}\left(\frac{W_2}{W_1 - W_x}\right)\right](W_1 - W_{o_1}) = \frac{PV}{NRT^2_2}\left(\frac{T_2 - T_1}{W_1 - W_x}\right)\frac{dW_x}{dt} +$$

$$(W_x - W_{o_x})\left[\frac{T_2 - T_1}{2T_2}\left(\frac{W_2}{W_1 - W_x}\right) - 1\right]$$

Since for any operating condition (W_x) is some fraction of (W_1), (W_x) may be written as $W_x = KW_1$ where $0 < K < 1$ [K is usually less than 0.3]. The above equation now becomes:

$$\frac{d^2(W_1 - W_{o_1})}{dt^2}\left[-\frac{IV}{NRT_2}\right] + \left[\frac{V}{NRT_2}\frac{dP_1}{dW_1} + \frac{PV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x} - I\frac{W_2}{P}\right]\frac{d(W_1 - W_{o_1})}{dt} +$$

$$\left[\frac{W_2}{P}\frac{dP_1}{dW_1} - 1 + \frac{(T_2 - T_1)}{2T_2}\left(\frac{W_2}{W_1 - W_x}\right)\right](W_1 - W_{o_1}) = \frac{KPV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x}\frac{d(W_1 - W_{o_1})}{dt} +$$

$$K(W_1 - W_{o_1})\left[\frac{T_2 - T_1}{2T_2}\left(\frac{W_2}{W_1 - W_x}\right) - 1\right]$$

Again collecting terms

$$\frac{d^2(W_1 - W_{o_1})}{dt^2}\left[-\frac{IV}{NRT_2}\right] + \frac{d(W_1 - W_{o_1})}{dt}\left[\frac{V}{NRT_2}\frac{dP_1}{dW_1} + \frac{PV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x} - \frac{IW_2}{P} - \frac{KPV}{NRT^2_2}\frac{(T_2 - T_1)}{W_1 - W_x}\right] +$$

$$(W_1 - W_{o_1})\left[\frac{W_2}{P}\frac{dP_1}{dW_1} - 1 + \frac{(T_2\ T_1)}{2T_2}\left(\frac{W_2}{W_1 - W_x}\right) - \left(\frac{KW_2}{W_1 - W_x}\right)\frac{(T_2 - T_1)}{2T_2} + K\right] = 0$$

or

$$\frac{d^2(W_1 - W_{o_1})}{dt^2} + \left[\frac{W_2 nRT_2}{PV} + \frac{KP}{IT_2}\,\frac{(T_2 - T_1)}{W_1 - W_x} - \frac{P}{IT_2}\,\frac{(T_2 - T_1)}{W_1 - W_x} - \frac{1}{I}\,\frac{dP_1}{dW_1}\right]\frac{d(W_1 - W_{o_1})}{dt} +$$

$$\left[\frac{nRT_2}{IV}\left(1 - \frac{W_2}{P}\,\frac{dP_1}{dW_1}\right) + \frac{nR}{IV}\,\left(\frac{KW_2}{W_1 - W_x}\right)\frac{(T_2 - T_1)}{2} - KT_2\right](W_1 - W_{o_1}) = 0 \;.\;.\; (24)$$

This integrates to

$$W_1 - W_{o_1} = C_1\,\varepsilon^{\left(-\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 - B}\right)t} + C_2\,\varepsilon^{\left(-\frac{\alpha}{2} - \sqrt{\left(\frac{\alpha}{2}\right)^2 - B}\right)t} \;.\;.\;.\;.\; (25)$$

where $\alpha = \dfrac{W_2 nRT_2}{PV} + \dfrac{KP}{IT_2}\,\dfrac{(T_2 - T_1)}{W_1 - W_x} - \dfrac{P}{IT_2}\,\dfrac{(T_2 - T_1)}{W_1 - W_x} - \dfrac{1}{I}\,\dfrac{dP_1}{dW_1}$ . . . . (26)

and $B = \dfrac{nRT_2}{IV}\left(1 - \dfrac{W_2}{P}\,\dfrac{dP_1}{dW_1}\right) + \dfrac{nR}{IV}\left\{\left(\dfrac{KW_2}{W_1 W_x}\right)\dfrac{(T_2 - T_1)}{2} - KT_2\right\}$ . . . . (27)

Equation (24) is a general stability equation which should apply with acceptable accuracy if the magnitude of the transient disturbance is not too great. The condition for any disturbance to be damped out is for (α) to remain a plus number, for if (α) becomes negative a disturbance will be amplified.

Instability starts when $\alpha = 0$

$$\frac{W_2 nRT_2}{PV} + \frac{KP}{IT_2}\,\frac{(T_2 - T_1)}{W_1 - W_x} - \frac{P}{IT_2}\,\frac{(T_2 - T_1)}{W_1 - W_x} - \frac{1}{I}\,\frac{dP_1}{dW_1} = 0 \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (28)$$

If we assume, for purposes of simplification, that the air-fuel ratio is large enough so that $W_2 \approx W_1 - W_x$, equation (28) may be written

$$\frac{W_2 nRT_2}{PV} + \frac{KP}{IT_2}\,\frac{(T_2 - T_1)}{W_2} - \frac{P}{IT_2}\,\frac{(T_2 - T_1)}{W_2} - \frac{1}{I}\,\frac{dP_1}{dW_1} = 0$$

from which

$$\frac{dP_1}{dW_1} = \frac{W_2 nIRT_2}{PV} + \frac{KP}{W_2}\,\frac{(T_2 - T_1)}{T_2} - \frac{P}{W_2}\,\frac{(T_2 - T_1)}{T_2}$$

or

$$\frac{dP_1}{dW_1} = \frac{W_2 nRT_2}{PV} + \frac{P}{W_2}\,\frac{(T_2 - T_1)}{T_2}(K - 1) = \frac{W_2 nRT_2}{PV} - \frac{P}{W_2}\,\frac{(T_2 - T_1)}{T_2}(1 - K) \ldots (29)$$

Thus for stability

$$\frac{dP_1}{dW_1} < \frac{W_2 nRT_2}{PV} - \frac{P}{W_2}\,\frac{(T_2 - T_1)}{T_2}(1 - K)$$

or since

$$1 - K = 1 - \frac{W_x}{W_1} = \frac{W_1 - W_x}{W_1} = \frac{W_2}{W_1},$$

instability starts when

$$\frac{dP_1}{dW_1} > \frac{W_2 nRT_2}{PV} - \frac{P}{W_1}\,\frac{(T_2 - T_1)}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

For the case of no burning $T_2 = T_1$ and $n = 1.0$ and from equation (26)

$$\alpha = \frac{W_2 RT_1}{PV} - \frac{1}{I}\left|\frac{dP_1}{dW_1}\right|_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

A direct evaluation of the inertia effect (I) for the system would be difficult to calculate. However, if the $\left(\frac{dP_1}{dW_1}\right)$ slope at which cold instability or cold pulsation starts is known, there appears to be no reason why the inertia of the system should be changed with combustion as long as the flow and the pressure are essentially the same. Equation (31) gives the value of (I) as

$$I = \frac{PV}{W_2 RT}\left|\frac{dP_1}{dW_1}\right|_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Substituting this value of (I) in equation (30) gives the condition for instability as

$$\frac{dP_1}{dW_1} = \frac{(W_2 nRT_2)}{PV}\,\frac{(PV)}{W_2 RT}\left|\frac{dP_1}{dW_1}\right|_c - \frac{P}{W_1}\,\frac{(T_2 - T_1)}{T_2}$$

or

$$\frac{dP_1}{dW_1} = \frac{nT_2}{T_1}\left|\frac{dP_1}{dW_1}\right|_c - \frac{P}{W_1}\,\frac{(T_2 - T_1)}{T_2} \quad \ldots\ldots\ldots\ldots\ldots (33)$$

or

$$\frac{W_1}{P}\,\frac{dP_1}{dW_1} = \frac{nT_2}{T_1}\left|\frac{W_1}{P}\,\frac{dP_1}{dW_1}\right|_c - \frac{T_2 - T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots (34)$$

Since this equation is exactly the same as the equation developed by Professor Bailey for conditions with no bleed off (K = 0), it appears that bleed off has no effect on the point where instability starts.

With wet compression, the air flow (W_1) is increased by the amount of moisture added (W_v). Equation (33) now becomes

$$\frac{dP_1}{d(W_1 + W_v)} = \frac{nT_2}{T_1}\,\frac{dP_1}{d(W_1 + W_v)}\Bigg|_c - \frac{P}{W_1 + W_v}\,\frac{(T_2 - T_1)}{T_2} \quad \ldots\ldots (35)$$

but since W_v = constant, $d(W_1 + W_v) = dW_1$ and equation (35) may be written as

$$\frac{dP_1}{dW_1} = \frac{nT_2}{T_1}\,\frac{dP_1}{dW_1}\Bigg|_c - \frac{P}{W_1 + W_v}\,\frac{(T_2 - T_1)}{T_2} \quad \ldots\ldots\ldots\ldots (36)$$

Equation (36) indicates that the effect of wet compression is to make $\left(\frac{dP_1}{dW_1}\right)$ more positive at the point of instability by decreasing the second term on the right side of the equation. Also, (T_1) the temperature of the air from the compressor is reduced by wet compression. This has the effect of increasing the first term on the right side of the equation and further reducing the second term, making dP_1/dW_1, even more positive. Thus, theoretically, the effect of wet compression would be to increase the range of the compressor.

WET COMPRESSION FOR USE WITH PNEUMATIC TOOLS

When a mixture of saturated air and moisture enters a compressor, the evaporated vapor behaves like any gas in that it is compressed with the air. When this compressed mixture is used in any mechanism where it is expanded to a temperature lower than the compressor discharge temperature, the ability of the air to contain the moisture is decreased and the moisture will condense out. If the saturated air is accelerated through a nozzle it is both reduced in static temperature and decreased in pressure, either of which will cause the air to become supersaturated and the vapor to condense out in the form of droplets. In a gas turbine power plant, the air remains at a temperature above the compressor discharge temperature until after the useful work has been extracted from it and the possibility of trouble from condensed moisture is remote. There remains, of course, the possibility of condensation of moisture due to centrifugal action, but this can be avoided in a well designed plant.

If, as in the case of pneumatic tools, the air is expanded to a temperature lower than the compressor discharge temperature, the moisture must be removed before the air reaches the tools. This means that the work required to compress the vapor is lost since the vapor is no longer a part of the working medium and work cannot be extracted from it. However, since the moisture is removed in an aftercooler, the removal takes place at nearly constant pressure, so that with wet compression, an increased pressure is obtained for a given amount of compression work or conversely for a required pressure to operate the pneumatic tools, the amount of compression work is reduced by wet compression.

For example, suppose a certain type of pneumatic tool requires

120 p.s.i. for operation. This is a pressure ratio of 8.18. For wet compression to achieve this pressure ratio, it will take .12 lbs of moisture per lb dry air and will result in a discharge temperature of about 227 deg F. The total work required to compress 1.120 lbs of mixture to a pressure of 8.18 atmosphere is given by $C_p dT + W_{v_1} h_g - W_{v_1} h_{g1}$ or about 130 BTU per pound of air. For an ideal dry compression to achieve a pressure ratio of 8.18, would result in a temperature of $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} =$ 980 deg R. The ideal dry compression work is then $C_p dT = 106$ BTU/lb air. Giving the compressor the benefit of an efficiency as high as .8, the actual dry compression work would be 133 BTU per lb air. Thus wet compression would save about 3 BTU for each pound of air used. In large capacity plants, this would amount to a considerable saving and would justify the additional initial cost of an aftercooler to remove the moisture added by wet compression.

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